Use of Graph Theory to Quantify Resilience in Multimodal Transportation Systems

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Graph Theory

Graph theory is particularly useful in studying the resilience transportation networks, because a network consists of a set of nodes – points representing a piece of infrastructure, such as a bridge or airport – and a set of links (roads, flights, routes, etc.) that connect them. These nodes and links can be easily analyzed through matrix analysis using matrices that describe various properties of the network. The network properties examined in this study are summarized in Table 1, but for the network ultimately modeled, average shortest path distance and diameter will be the primary network properties compared.

Table 1. Summary of Example Network Properties [1]

Metric	Calculated for	Definition	Equation
Link Density	Network	The fraction between the total and the maximum number of links	$\frac{2m}{n\,(n-1)}$
Average Node Degree		The average value of the node-degree distribution	$\frac{2m}{n}$
Average Shortest Path Distance		Average value of the distances between all pairs of nodes	$\sum_{i}\sum_{j}\frac{l\,(i,j)}{n\,(n-1)}$
Diameter		Maximum shortest path distance between all pairs of nodes	$\max(\,L(i,j)\,)$
Betweenness Centrality	Node	Proportion of shortest paths that run through a given node	$\frac{1}{(n-1)(n-2)}\sum_{j,k\neq i} a_{jk(i)}$

When using graph theory to study a network's resilience, the same basic method applies no matter the type of network. First, selected network properties are calculated. Then, nodes are removed from the system one at a time and the same network properties are recalculated. Nodes are removed from the network using one of two node removal strategies: a random node removal strategy (RNRS), and a targeted node removal strategy (TNRS). RNRS simulates disturbances to the system that have the same likelihood of occurring at any point in the system, like weather events or power outages. TNRS simulates deliberate attacks to important points in the network. In this study, two methods for TNRS were plotted, TNRS-a- node removal base on betweeness centrality, and TNRS-b, where betweeness centrality was recalculated after each node removal.

Methodology/Results

The graph theory methodology described in Aderinlewo and Attoh-Okine [1] was applied to the highway network system of Albemarle County, except that in this study, traffic volumes were added as weights between links in the system. The mapped system is included in Figure 1:

Weights were assigned using annual average daily traffic (AADT) counts for sections of the roadway. Two methods were used to accomplish this. The first altered the weight value directly proportional to AADT, and the second considered the order of magnitude of the AADT. Average shortest path distance and diameter were calculated for an unweighted scenario and for each of the aforementioned weighting schemes (WN1, WN2) for the highway network.

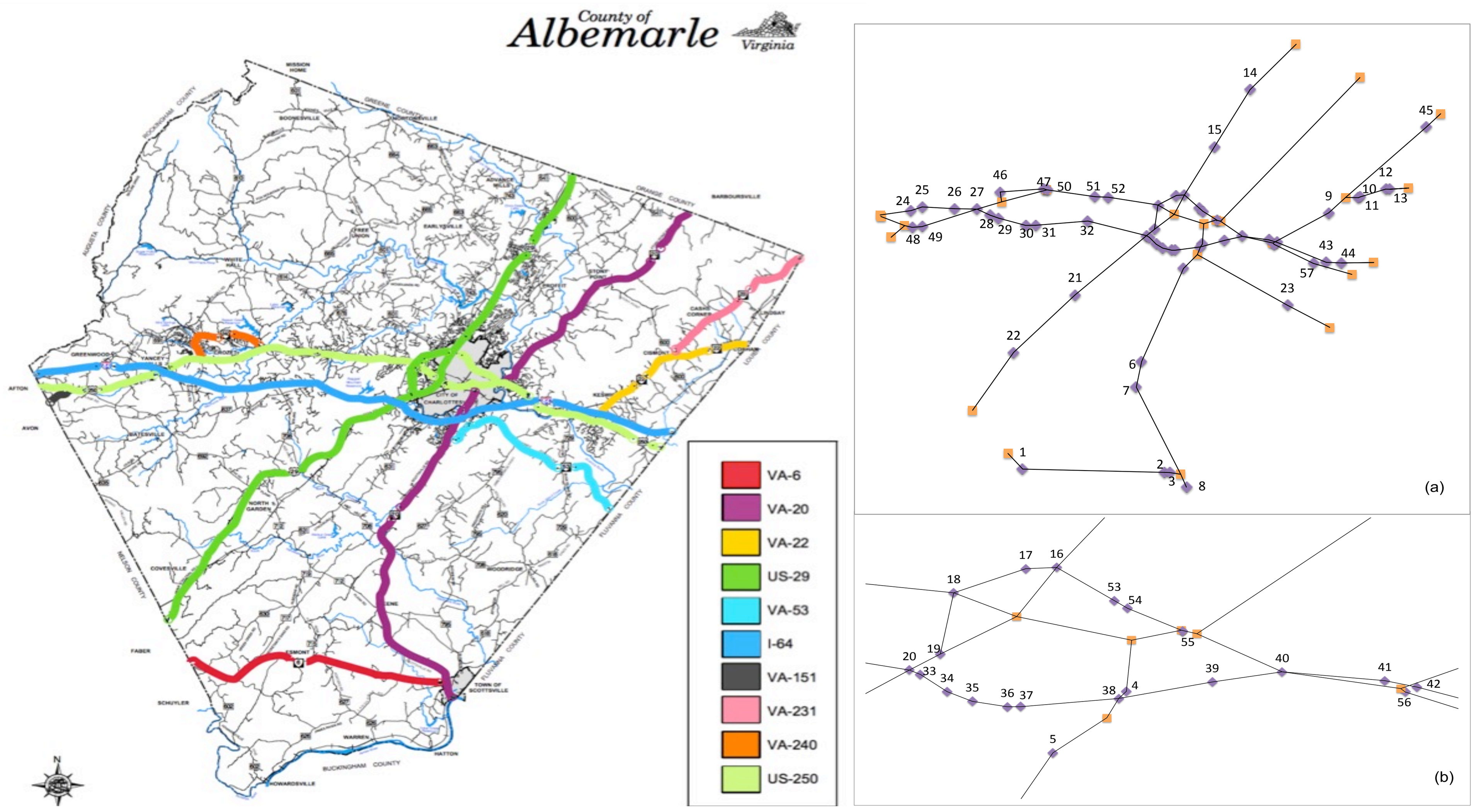

Figure 1. Map of Albemarle Roads included with a) network map and b) detail of Charlottesville.

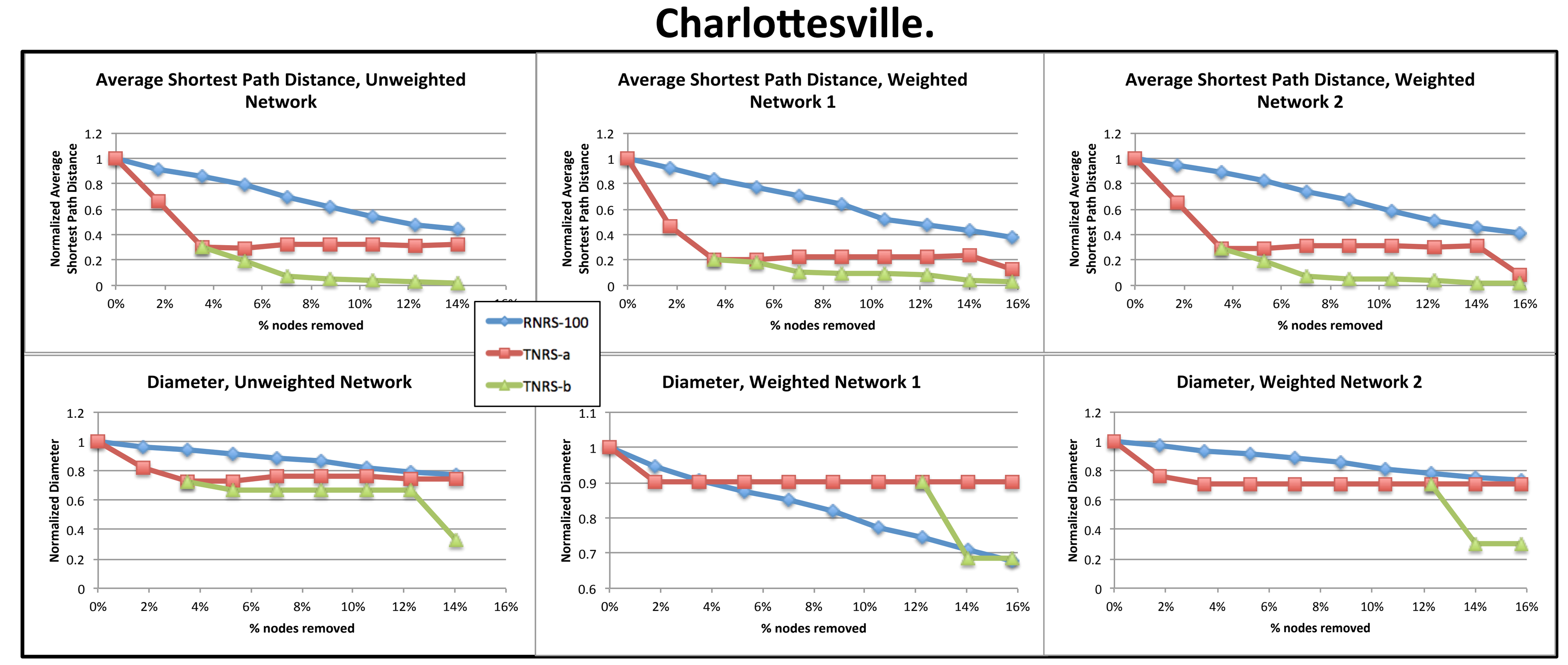

Figure 2. Comparison of Metrics for Weighting Schemes

Next, nodes were removed using RNRS and TNRS. For RNRS, a random number generator determined the order of node removal, and 100 removal runs were conducted and averaged. After each node removal, network properties were recalculated. Figure 2 shows a comparison of the average shortest path distance and diameter plots for all three scenarios - the unweighted network and two weighting schemes (WN1 and WN2).

Conclusions

The unweighted network's resilience index was found to be 14%. The direct proportionality method, used in WN1, and the order of magnitude method, used in WN2, both led to average shortest path plots that did not intersect at all, indicating that the network is not resilient once any node is removed. The fact that both weighting methods effectively lowered the resilience index of the network highlights the importance of adding weights to links in analyses such as the one performed here: it offers a more complete picture of the system, and failing to include it can inflate the network's resilience index.

References

[1] Aderinlewo, Olufikayo and Attoh-Okine, Nii. Assessment of a Transportation Infrastructure System Using Graph Theory. *Acta Technica Napocensis: Civil Engineering & Architecture*. Vol. 56, No. 1, 2013, pp. 12-24.